Innermost stable circular orbits around magnetized rotating massive stars

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In 1998, Shibata and Sasaki [Phys. Rev. D **58**, 104011 (1998)] presented an approximate analytical formula for the radius of the innermost stable circular orbit (ISCO) of a neutral test particle around a massive, rotating, and deformed source. In the present paper, we generalize this expression by including the magnetic dipole moment. We show that our approximate analytical formula is accurate enough by comparing it with the six-parametric exact solution calculated by Pachón *et al.* [Phys. Rev. D **73**, 104038 (2006)] along with the numerical data presented by Berti and Stergioulas [Mon. Not. R. Astron. Soc. **350**, 1416 (2004)] for realistic neutron stars. As a main result, we find that in general, the radius at ISCO exhibits a decreasing behavior with an increasing magnetic field. However, for magnetic fields below 100 GT the variation of the radius at ISCO is negligible and hence the nonmagnetized approximate expression can be used. In addition, we derive approximate analytical formulas for angular velocity, energy, and angular momentum of the test particle at ISCO.

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I. INTRODUCTION

The discovery of quasiperiodic oscillations (QPOs) with frequencies around 1 kHz from several low-mass X-ray binaries [1] has been increasing the interest in the detailed theory of disk accretion onto neutron stars. Several authors have suggested that at least some of the kHz QPOs may be related to the Kepler frequency at the innermost stable circular orbit (ISCO) of the accretion disk around a neutron star (see, e.g., [1,2]). Stergioulas *et al.* [3] have suggested that the frequency of the corotating orbit at ISCO in a compact stellar remnant could be determined through X-ray observations of low-mass X-ray binaries and it could be used to constrain the equation of state (EOS) of ultra-dense matter. Morsink and Stella [4] have remarked the central role of ISCO in the relativistic precession of orbits around neutron stars and Bulik *et al.* [5] have shown that observations are consistent with the assumption that the maximum-frequency QPOs occurs at the ISCO. The last statement could be used to test general relativity (GR) in the strong-field regime around accreting neutron stars, or even to measure the stellar mass by directly comparing the highest frequency manifest in the X-ray flux with the relativistic formula for the orbital frequency in the ISCO orbit [6].

On the other hand, the study of the structure and dynamics of neutron stars endowed with a magnetic field in GR is an active, interesting, and challenging theoretical issue. The influence of a magnetic field on the properties of a neutron star rotating at the Kepler frequency has been shown in Ref. [7]. In Ref. [8], Broderick *et al.* have studied the implications of very strong magnetic fields on the structure of neutron stars; in particular, Cardall *et al.* [9] have indicated how magnetic field affects the maximum mass of stars. In Ref. [10], the ellipticity of the deformed star due both to the rotation and the magnetic field is calculated, and these two effects are compared to each other within GR. In addition, the formulation of deformation of relativistic stars due to the magnetic stress, considering the magnetic fields as perturbations from spherical stars, has also been studied in [11] by means of an analytical treatment assuming weak magnetic fields compared to gravity. The quadrupole deformation of magnetized Newtonian stars was discussed by Chandrasekhar and Fermi [12] and Ferraro [13]. The GR approach was done fully numerically by Bonazzola and Gourgoulhon [14] and Bocquet *et al.* [15], who pointed out that deformations of the stars induced by magnetic fields become appreciable only for fields greater than 10 GT.

More than one decade ago, Shibata and Sasaki [16] (hereafter S&S) computed an approximate analytical formula for the radius at ISCO on massive rotating and arbitrarily deformed sources within GR. They considered the role of the quadrupole moment of mass in physics related to neutron stars (this fact has also been noted by other authors, see, for instance, [17] and references therein), by including multipolar moments of mass up to the 2^4-pole order in their calculations. A strong influence of the magnetic field on the structure of the neutron stars

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